Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 53248

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2019.

Third Semester

Civil Engineering

MA 6351 — TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to Mechanical Engineering (Sandwich)/Aeronautical Engineering/Agriculture Engineering/Automobile Engineering/Biomedical Engineering/Computer Science and Engineering/Computer and Communication Engineering/Electrical and Electronics Engineering/Electronics and Communication Engineering/Electronics and Instrumentation Engineering/Geoinformatics Engineering/Industrial Engineering/ Industrial Engineering and Management/Instrumentation and Control Engineering/Manufacturing Engineering/Marine Engineering/Materials Science and Engineering/Mechanical Engineering/Mechanical and Automation Engineering/Mechatronics Engineering/Medical Electronics/Petrochemical Engineering/Production Engineering/Robotics and Automation Engineering/Bio Technology/Chemical Engineering/Chemical and Electrochemical Engineering/Food Technology/Information Technology/Petrochemical Technology/Petroleum Engineering/Plastic Technology/Polymer Technology)

(Regulation 2013)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Form the partial differential equation by eliminating the arbitrary function f from $z = f\left(\frac{y}{x}\right)$.

2. Find the complete solution of the partial differential equation $\sqrt{p} + \sqrt{q} = 1$.

3. State Dirichlet condition for existence of Fourier series.

4. If $(\pi - x)^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{\cos nx}{n^2}$, in $0 < x < 2\pi$, then deduce the value of $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

5. Classify the following partial differential equation $u_{xx} + u_{xy} + u_{yy} = 0$.

6. What are the possible solutions of the one dimensional heat flow equation.

7. Find the Fourier sine transform of e^{-ax}.

8. Define self reciprocal function under Fourier transform with example.

9. Find the Z transform of a constant 'a'.

10. If $Z\{f(n)\} = \dfrac{z^2}{z^2+1}$, then find $f(0)$, using initial value theorem.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the singular solution of $z = px + qy + p^2q^2$. (8)

(ii) Solve $(D^2 - 2DD')z = x^3y + e^{2x}$. (8)

Or

(b) (i) Find the complete solution of $p^2 + q^2 = x^2 + y^2$. (8)

(ii) Find the general solution of $(y - z)p + (z - x)q = (x - y)$. (8)

12. (a) (i) Find the Fourier series of $f(x) = x^2$ in $-\pi < x < \pi$. (8)

(ii) Find the half range sine series expansion of $x(\pi - x)$ in $0 < x < \pi$. (8)

Or

(b) (i) Compute the first two harmonics of the Fourier series of $f(x)$ from the table given (8)

x	0	π/3	2π/3	π	4π/3	5π/3	2π
$f(x)$	1.0	1.4	1.9	1.7	1.5	1.2	1.0

(ii) Obtain the Fourier cosine series expansion of $f(x) = x$ in $0 < x < l$. (8)

13. (a) A tightly stretched siring with fixed end points $x = 0$ and $x = l$ is initially at rest in its equilibrium position. If it is set vibrating each point a velocity $3x(l - x)$, find the displacement of the string. (16)

Or

(b) A rectangular plate with insulated surface is bounded by the lines $x = 0$, $x = a$, $y = 0$ and $y = b$. The temperature along the edge $y = b$ kept at 100°C, while the temperature along the other three edges are at 0°C, find the steady - state temperature at any point in the plate. (16)

14. (a) Find the Fourier transform of $f(x)=\begin{cases}1-|x|, & if\,|x|<1\\ 0, & \text{otherwise}\end{cases}$. Hence deduce the values of

(i) $\int_0^\infty \frac{\sin^2 t}{t^2}\,dt,$

(ii) $\int_0^\infty \frac{\sin^4 t}{t^4}\,dt.$ (16)

Or

(b) (i) Find the Fourier transform of $e^{-a^2x^2}$, where $a>0$. (8)

(ii) Use transform methods to evaluate $\int_0^\infty \frac{dx}{(x^2+a^2)(x^2+b^2)}$. (8)

15. (a) (i) Find the inverse Z-transform of $\frac{z^2}{(z-a)^2}$ by using convolution theorem. (8)

(ii) Solve $u_{n+2}+6u_{n+1}+9u_n=2^n$ with $u_0=0$, $u_1=0$, by using Z-transforms. (8)

Or

(b) (i) Find Z-transform of $\frac{1}{n(n+1)}$. (8)

(ii) Find the inverse Z-transform of $\frac{z^2+z}{(z^2+1)(z-1)}$. (8)